



Title and Abstract

Title: *Awesome Mathematical Knitting Constructions*

Abstract:

This talk will explore physical constructions of two dimensional manifolds (Torus, Möbius, Klein bottle) created through knitting by using their inherent properties.

Although no physical knitting experience is required or provided, I will supplement the talk with a selection of knitted projects as examples of each concept.

Awesome Reversible Knitting

Mathematical

Karen Little


BMath (OR), UWaterloo

@AwesomeReversibleKnitting

on YouTube



The AMK
Lady?



I'm Jaap

Outline

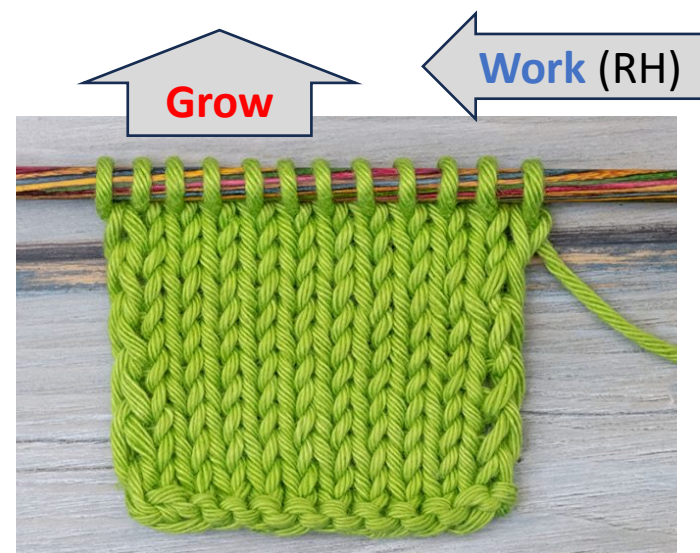
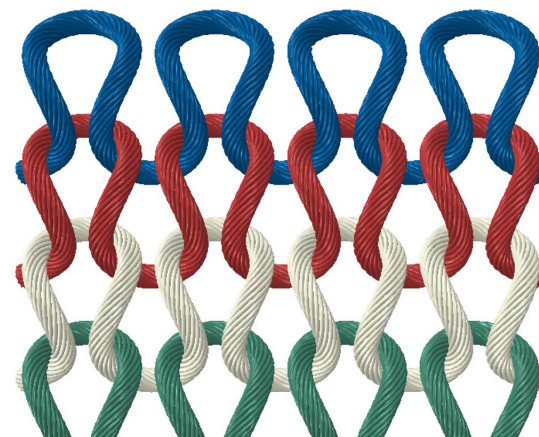
- (Very) short explanation of Knitting
- Torus
 - Two constructions
- Möbius
 - Two constructions
- Klein Bottle
 - At least four constructions

- Who is familiar with these surfaces?

I know, pick me!

What is Knitting?

- A surface of interlocking loops of filaments or strands (yarn, thread, string, wire etc)
 - Isomorphic to original yarn
- **Worked** horizontally, **grows** vertically
 - Can be worked **flat** (reverse rows), by turning at the end of the row
 - Can be worked **circular** (spiral), by connecting to the beginning



Seems simple!

Torus

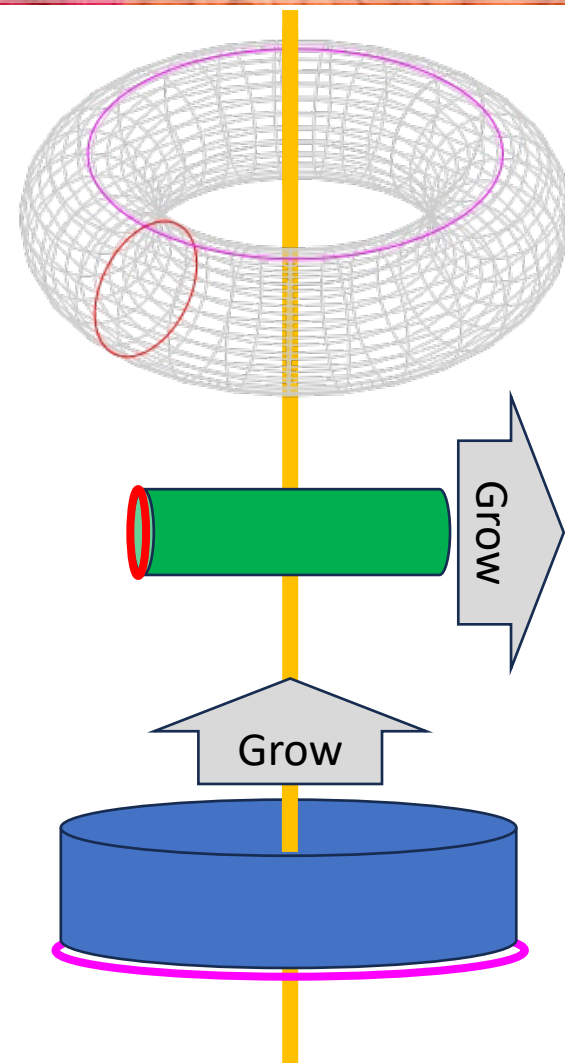
∴ Torus is a circle of a circle

1. Toroidal construction

- Start with (cast on) a **small** circle
- Knit a **long** section **circular**
- Join (graft) the end to the beginning
 - End to end, parallel to central **axis**

2. Poloidal construction

- Start with (cast on) a **large** circle
- Knit a **short** section **circular**
- Join (graft) the end to the beginning
 - In parallel circles, perpendicular to **axis**

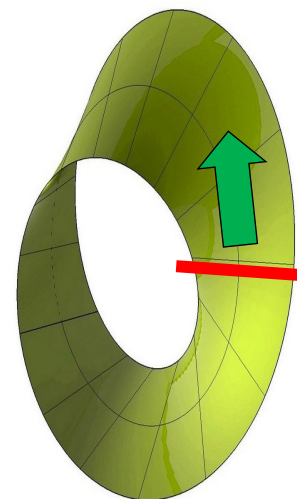


Watch the videos!

Möbius

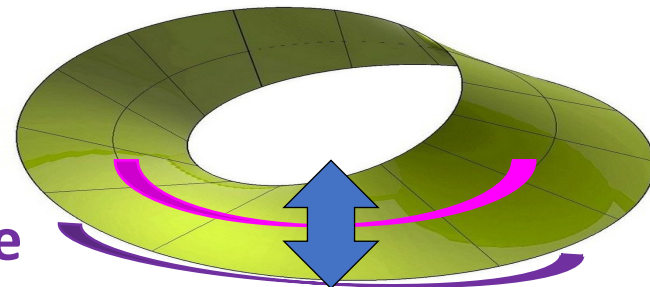
1. Transverse construction (:: one side)

- Start with (cast on) a **small** width
- Knit a **long** section **flat**
- Add a half twist (with L or R chirality)
- Join (**graft**) the end to the beginning



2. Helical construction (:: one edge)

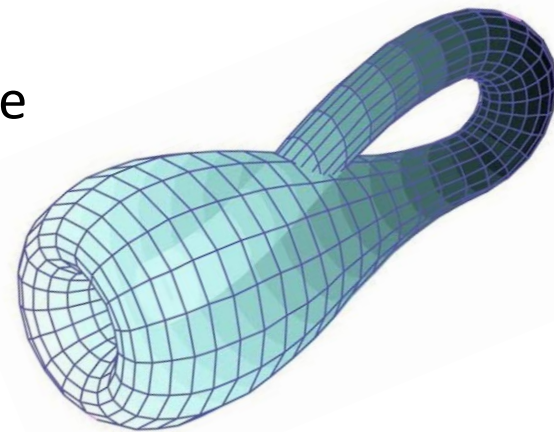
- Start with a **long** double sided cast on (at the **midline**)
- Knit into the back side of the cast on to add half twist (with L or R chirality)
- Knit a **short** section **circular**, one möbius round is 720°
- Finish by casting off the single **edge**



Klein means Little!

Klein Bottle

- Like the Torus, the Klein bottle is a **closed** surface (ie has no edge or boundary)
- Like the Möbius, the Klein bottle is a **non-orientable** surface (ie has only one side)
- Specifically, the Klein bottle is a **closed**, **non-orientable**, two dimensional manifold
 - ie Surface without edges and only one side
 - Immersed in \mathbf{R}^3 , requires self intersection
- Four Klein bottle homotopy classes
 - Achiral (Left/Right and Right/Left)
 - Chiral (Left and Right)
- Many constructions of Klein bottles of various classes



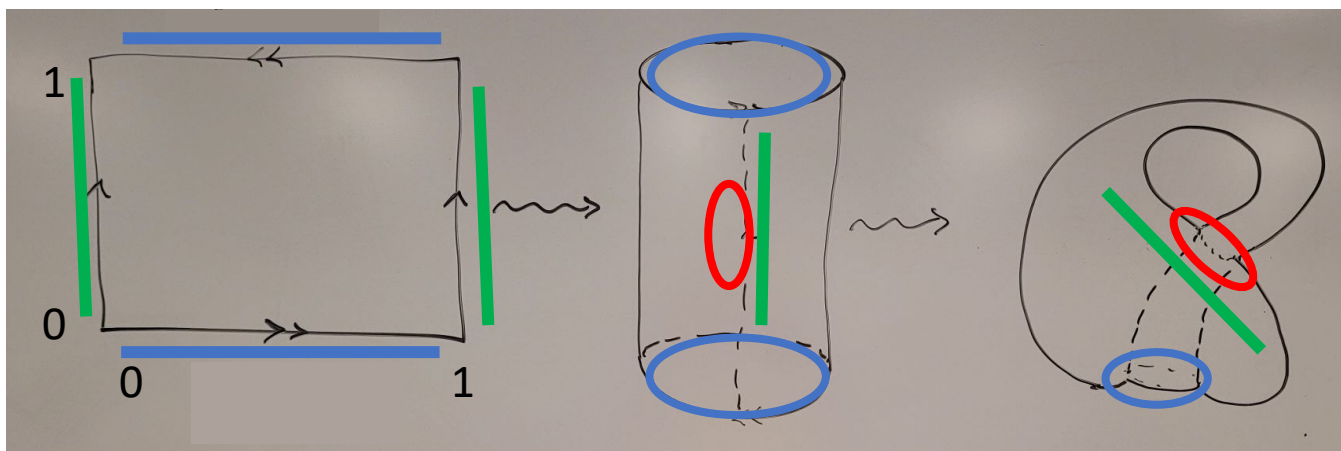
Thanks to KB+

Klein Bottle

$\because (0, y) \sim (1, y)$ for $0 \leq y \leq 1$ and $(x, 0) \sim (1 - x, 1)$ for $0 \leq x \leq 1$

1. Toroidal or Transverse construction

- Create a **cylinder** with a **hole** on the side
- Pass the end through the **hole** into the fourth dimension
- **Graft** the end to the beginning
- Create achiral or chiral Klein bottles



Who's convinced?

Klein Bottle

:: Achiral Klein bottle can be split into two Möbius

2. Double Möbius construction

- Create two helical Möbius with **opposite** chirality
- Graft the single edge of Möbius to itself (with a hole)



Swatches: G/W connected

Single yarn!

Klein Bottle

:: Achiral Klein bottle can be split into two Möbius

3. Helical construction

- Create a helical Möbius (midline cast on, any chirality)
- Create the hole along the edge
 - Cast off a section of stitches
 - Create a new cast on for the section in the next Möbius round
- Rotate into the fourth dimension until live edges are adjacent for the entire length before attaching (full twist)
- Continue knitting the second Möbius (opposite chirality)
- Graft the second Möbius together at its midline



What about 2?

Klein Bottle

∴ Chiral Klein bottle can be created from a single Möbius

4. Single Möbius construction

- Create a helical Möbius (midline cast on, any chirality)
- Cast off a small section of the edge
- Fold the Möbius at the midline and graft the rest of the edge to itself
- Cast off other side of the hole
- Chirality of the Möbius matches the Klein bottle



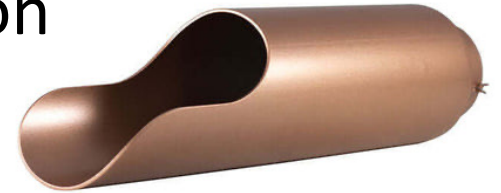
That's cheating!

Klein Bottle

∴ A hole is required for self-intersection

5. Scupper Construction

- Knit a small section flat (**scupper**)
- Join to start knitting circular and knit a **long** section
- Separate and knit a small section flat (**scupper**)
- Graft the scuppers to each other
- If the scuppers are in **opposite** orientations
 - Graft the ends together
 - Creates an **achiral** Klein bottle
- If the scuppers are in the **same** orientation
 - Add a half twist (like for transverse Möbius)
 - Creates left or right **chirality** Klein bottle



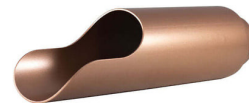
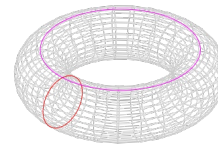
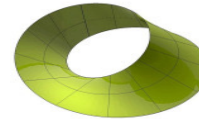
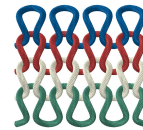
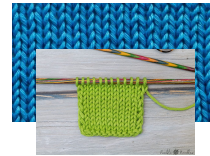
Next Steps

My Open Questions:

- For toroidal or transverse Klein bottle construction
- YES!* • Can a chiral Klein bottle be created by grafting with a 180° twist?
 - What about if the chiral twist is $\neq 180^\circ$? *?*
- How are C3 and C4 related?
 - *C3 creates an achiral KB, so after the first mobius is created the full twist sets up the second (opp chirality) mobius.*
 - *C4 creates a chiral KB, so after the mobius is complete, the KB hole is cast off on each side, maintaining the half twist*
- How can other parametrizations be constructed?
 - Figure 8 immersion
- What about the Real Projective Plane?

References

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- <https://en.wikipedia.org/wiki/Torus>
- https://en.wikipedia.org/wiki/Klein_bottle
- https://en.wikipedia.org/wiki/Fundamental_polygon
- <https://majesticwaterspouts.com/collections/mid-sized-water-scuppers>
- <https://youtube.com/@AwesomeReversibleKnitting>





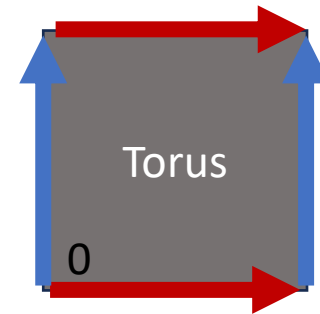
My Background

- University of Waterloo, BMath (Operations Research)
- 25 years Supply Chain management
 - Forecasting, (future) order and inventory management, capacity planning, production scheduling, transportation routing, measurement, reporting and education
- *@AwesomeReversibleKnitting* YouTube channel
 - Reversible stitch patterns and projects
 - Creative and educational videos
- Thank you Alexanna (Xanna) for asking me!
- Thank you Kristine for pointing me to Klein bottles!

Fundamental Polygons

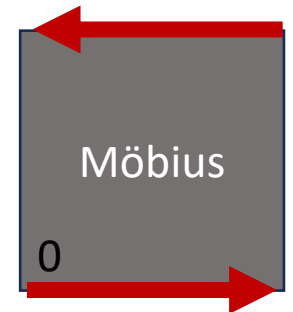
- Torus

- $(x, 0) \sim (x, 1)$ for $0 \leq x \leq 1$ and $(0, y) \sim (1, y)$ for $0 \leq y \leq 1$



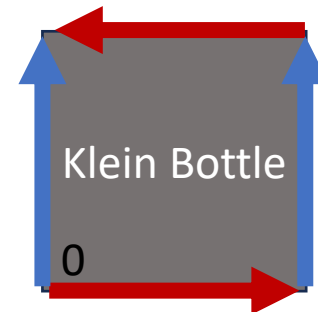
- Möbius

- $(x, 0) \sim (1 - x, 1)$ for $0 \leq x \leq 1$



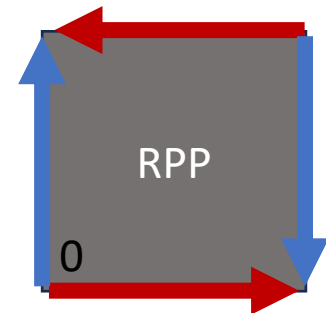
- Klein Bottle

- $(x, 0) \sim (1 - x, 1)$ for $0 \leq x \leq 1$ and $(0, y) \sim (1, y)$ for $0 \leq y \leq 1$



- Real Projective Plane

- $(x, 0) \sim (1 - x, 1)$ for $0 \leq x \leq 1$ and $(0, y) \sim (1, 1 - y)$ for $0 \leq y \leq 1$



My Conventions

In knitted fabric, you cannot determine based on the structure alone if the stitch was created as a Knit or Purl

	1. How it's worked	2. How it looks (on a given side, regardless of how it was worked)
Stitch	Knit stitch	Knit <i>looking</i> stitch
Stitch Pattern Eg: All stitches the same	Garter (flat) Stockinette (circ)	Stockinette (flat) Garter (circ)
Row Repeat	Odd (Naturally reversible)	Even (2*Odd)

Sets for Knitting

Swatch

(Piece of Fabric)

Single Stitches
(Eg: K, P,
K2tog, YO)

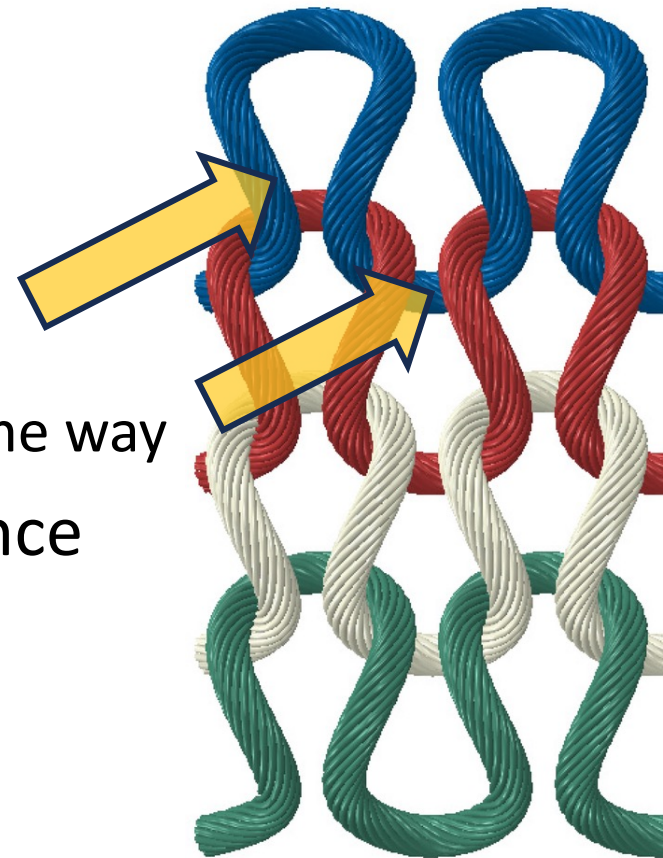
Compound Stitches
Integrated combos
(Eg: Cables,
K2tog K1st again)

Stitch Patterns (Tilings)
Segregated combos, often named
(Eg: Farrow Rib, Moss)

Edges
(Eg: Cast on/off
Selvedges)

Physics of Knitted Fabric

- Why does some knitted fabric curl?
 - Eg Stockinette (based) curls two ways
- Knit and Purl imbalance
 - Loops (stitches) all face the same way
 - Legs (between stitches) all pull the same way
- Reversible stitch patterns have balance
 - Forces are in equilibrium
- Except the Mesh Stitch
 - One row repeat that curls diagonally!





Physics of Knitted Fabric

- Knits adjacent and stacked with Purls
 - Knit stitch sits in front of Purl when adjacent
 - Knit stitch sits under Purl when stacked
- Changes the look of the fabric
 - Eg Welts, Farrow Rib
- Changes the gauge of the fabric
 - Yellow swatch has the same number of stitches in each row, and rows in each “half”

Gauging and Ratios for sizing

- Knitters often make a practice or gauge swatch before starting a large project
 - Measure the stitch and row count (St/cm and rs/cm)
 - Varies for each stitch pattern
 - Varies based on yarn weight, needle size, tension and yarn fibre
 - Calculate the number of cast on stitches and row pattern repeats based on the ratios to get desired finished size
- Ratios are also used to determine the rate of stitch increase or decrease to get a specific slope
 - Used for shaping clothing like triangular shawls and hats
 - Small slope, increase faster; big slope, increase slower